Limits Of Beach And Dune Erosion In Response To Wave Runup From Large-Scale Laboratory Data

by

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Date of Approval:
April 30, 2008

Keywords: beach erosion, nearshore sediment transport, swash excursion, wave breaking, physical modeling, surf zone processes, coastal morphology

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Acknowledgements

The support and guidance of those who have contributed to the successful completion of this thesis have been greatly appreciated. Most importantly, I would like to sincerely thank Dr. Ping Wang for his knowledge, instruction, and guidance during this project. From guidance during my undergraduate Honors Thesis through the present, Dr. Wang has been the most important and influential person in my academic, educational, and personal advancements. I would also like to extend sincere thanks to Dr. Nicole Elko, whose guidance through my thesis lead to a more comprehensive understanding of coastal environments and processes. Her time and dedication to my academic advancement played an instrumental role in the development and successful completion of this thesis.

I would also like to thank Dr. Nicholas Kraus for the SUPERTANK dataset and significant contributions to the study including insights into the broader implications of swash processes in many different coastal applications. Thanks to my other committee members, Drs. Chuck Connor and Rick Oches, whose contribution guided me towards a more comprehensive analysis of this topic.

Finally, thanks to my family and friends who have supported me during my educational endeavors with constant encouragement to pursue my academic and personal aspirations and dreams. Thank you all.
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Limits of Beach and Dune Erosion in Response to Wave Runup from Large-Scale Laboratory Data

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Abstract

The SUPERTANK dataset is analyzed to examine the upper limit of beach change in response to elevated water level induced by wave runup. Thirty SUPERTANK runs are investigated, including both erosional and accretionary wave conditions under random and monochromatic waves. Two experiments, one under a spilling and one under a plunging breaker-type, from the Large-Scale Sediment Transport Facility (LSTF) are also analyzed. The upper limit of beach change approximately equals the maximum vertical excursion of swash runup. Exceptions to this direct relationship are those with beach or dune scarps when gravity-driven changes, i.e., avalanching, become significant. The vertical extent of wave runup, $R_{\text{max}}$, above mean water level on a beach without a scarp is found to approximately equal the significant breaking wave height, $H_{bs}$. Therefore, a simple formula $R_{\text{max}} = H_{bs}$ is proposed. The linear relationship between maximum runup and breaking wave height is supported by a conceptual derivation. This predictive formula reproduced the measured runup from a large-scale 3-dimensional movable bed physical model. Beach and dune scarps substantially limit the uprush of swash motion, resulting in a much reduced maximum runup. Predictions of wave runup are not improved by including a slope-dependent surf-similarity parameter. The limit of
wave runup is substantially less for monochromatic waves than for random waves, attributed to absence of low-frequency motion for monochromatic waves.
**Introduction**

Accurate prediction of the upper limit of beach change is necessary for assessing and predicting dramatic morphological changes accompanying storms. It is particularly important for coastal management practices, such as designing seawalls, nourishment profiles, and assessing potential coastal damages. The upper limit of beach change is controlled by wave breaking and the subsequent wave runup. During storms, wave runup is superimposed on elevated water levels (due to storm surge). Wang et al. (2006) found that the highest elevation of beach erosion induced by Hurricane Ivan in 2004 extended considerably above the measured storm-surge level, indicating that the wave runup played a significant role in the upper limit of beach erosion. The limit of wave runup is also a key parameter for the application of the storm-impact scale by Sallenger (2000). The Sallenger scale categorizes four levels of morphological impact by storms through comparison of the highest elevation reached by the water (storm surge and wave runup) and a representative elevation of barrier island morphology (e.g. the crest of the foredune ridge). In addition, accurately assessing wave runup has numerous engineering applications, including design waves for overtopping of seawalls, breakwaters and jetties, and elevation of berm height for beach nourishment (Komar, 1998). Therefore, quantification of wave runup and its relationship to the upper limit of beach-profile change are essential for understanding and predicting beach and dune changes. Limit of
runup typically serves as a crucial parameter for the modeling of coastal morphology changes.
Wave runup is composed of wave setup and swash runup, defined as a super-elevation of the mean water level and fluctuations about that mean, respectively (Guza and Thornton, 1981; Holman and Sallenger, 1985; Nielsen, 1988; Yamamoto et al., 1994; Holland et al., 1995). Wave setup can be further defined as a seaward slope in the water surface that provides a pressure gradient or force balancing for the onshore component of the radiation stress induced by the momentum flux of waves (Komar, 1998). The swash runup component of wave runup is defined as the upper limit of swash uprush. The swash uprush is strongly modulated by low-frequency oscillations often referred to as infragravity waves with periods of at least twice the peak incident wave periods (Komar, 1998).

Previous studies on the limits of wave runup were primarily based on field measurements at a few locations resulting in several formulas developed to predict wave setup and runup. An early formula for wave setup slope was based on a theoretical derivation for sinusoidal, or monochromatic, waves on a uniformly sloping beach (Bowen et al., 1968):

\[
\frac{\partial \bar{\eta}}{\partial x} = -K \frac{\partial h}{\partial x} \quad K = (1 + 2.67 \gamma^{-2})^{-1}
\]

where \( h = \) still-water depth, \( \bar{\eta} = \) wave setup, \( x = \) cross-shore coordinate, \( \gamma = H/(\bar{\eta} + h) \) and \( H = \) wave height. The term \( \gamma \) essentially describes the relationship between the

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breaking wave height and its proportionality to local water depth. Based on both theory and laboratory measurements, the maximum set-up under a monochromatic wave, $\bar{\eta}_M$, was found to occur at the still-water shoreline (Battjes, 1974):

$$\frac{\bar{\eta}_M}{H_b} = 0.3\gamma$$

(2)

where $H_b =$ breaking wave height. The above equations concern only the setup component of the entire wave runup, not including the portion of elevated water level induced by swash runup. Taking the commonly used value of 0.78 for $\gamma$, the maximum setup yielded from the above equation is about 23% of the breaking wave height.

Based on field measurements conducted on dissipative beaches, Guza and Thornton (1981) found maximum setup at the shoreline, $\bar{\eta}_s$, to be linearly proportional to the significant deepwater wave height, $H_o$:

$$\bar{\eta}_s = 0.17H_o$$

(3)

Guza and Thornton (1982) found that the significant wave runup, $R_s$, (including both wave setup and swash runup) is also linearly proportional to the significant deep-water wave height:

$$R_s = 3.48 + 0.71H_o \quad \text{(units of centimeters)}$$

(4)

Comparing Eqs. (3) and (4), the entire wave runup is approximately 4 times the wave setup, i.e., swash runup constitutes a significant portion, approximately 75%, of the total elevated water level. According to Huntley et al. (1993), Eq. (4) is the best choice for predicting wave runup on dissipative beaches. Based on field measurements on highly dissipative beaches, Ruessink et al. (1998) and Ruggiero et al. (2004) also found linear relationships, but with different empirical coefficients.
Based on field measurements on both dissipative and reflective beaches, Holman (1986) and several similar studies (Holman and Sallenger, 1985; Ruggiero et al., 2004; Stockdon et al., 2006), argued that more accurate predictions can be obtained by including the slope-dependent surf-similarity parameter, $\xi$:

$$\xi = \frac{\beta}{(H_o / L_o)^{1/2}}$$

(5)

where $L_o =$ deepwater wavelength, and $\beta =$ beach slope. The surf similarity parameter has been broadly used to parameterize many nearshore and surf-zone processes including morphodynamic classification of beaches, wave breaking, breaker-type, the number of waves in the surf zone, and runup (Iribarren and Nogales, 1949; Battjes, 1971; Galvin, 1972; Battjes, 1974; Galvin, 1974; Holman and Sallenger, 1985). Holman (1986) emphasized the application of the surf similarity for setup and swash runup predictions. Holman found a dependence of the 2% exceedence of runup, $R_2$, on the deepwater significant wave height and the surf similarity parameter:

$$R_2 = (0.83\xi_o + 0.2)H_o$$

(6)

The 2% exceedence of runup refers to a statistic of runup measurements of which only 2% of the data exceeded. Stockdon et al. (2006) expanded upon the Holman (1986) analysis with additional data covering a wider range of beach slopes and developed a more complicated empirical equation:

$$R_2 = 1.1 \left[ 0.35\beta_f (H_o L_o)^{1/2} + \left[ H_o L_o (0.563\beta_f^2 + 0.004) \right]^{1/2} \right]^{1/2}$$

(7)

where $\beta_f =$ foreshore beach slope. Realizing the variability of beach slope in terms of both definition and measurement, Stockdon et al. (2006) defined the foreshore beach
slope as the average slope over a region of two times the standard deviation of continuous water level record.

Nielsen and Hanslow (1991) found a relationship between the surf similarity parameter and runup on steep beaches. However, for beaches with a slope less than 0.1, they suggested that the surf similarity parameter was not related to runup. A subsequent study by Hanslow and Nielsen (1993) conducted on the dissipative beaches of Australia found that maximum setup was not dependent on beach slope.

Except for the original derivation by Bowen et al. (1968), most predictive formulas for wave runup have been empirically derived based on field measurements over primarily dissipative beaches, with a limited number of measurements over reflective beaches. Based on a study by Guza and Thornton (1982), maximum swash excursion on beaches with wide surf zones was found to depend primarily on the infragravity modulation of swash, or the low-frequency oscillations. Because surf bore heights are depth limited, an increase in offshore wave height increases the distance the bore must travel further dissipating its energy, i.e., wave energy at the incident frequency has become saturated. In contrast, the energy in the low-frequency range increases shoreward due to non-linear energy transfer (Guza and Thornton, 1982). This modulation of infragravity swash motions has been investigated in several other studies (Holman and Guza, 1984; Howd et al., 1991; Raubenheimer et al., 1996). Energy transfer from incident frequencies to infragravity frequencies is non-linear and not well understood. Figure 1 illustrates the power density spectra of spilling and plunging breakers measured during the LSTF experiments at an offshore gauge and at a gauge near the shoreline. Most of the energy peaks within the high-frequency domain prior to wave
breaking. However, as the waves propagate onshore after breaking, much of the energy peaks in the low-frequency domain. Although the significance of infragravity motions is qualitatively recognized, existing theoretical and the empirical equations do not include infragravity wave parameters.

Figure 1. Power density spectra from the LSTF experiment under spilling and plunging breakers measured from an offshore gauge (above) and a gauge at the shoreline (below).
The definition of dissipative and reflective beaches was developed as a classification scheme for overall beach morphology (Short, 1979a, b; Wright et al, 1979a, b; Wright and Short, 1984). One of the key factors to the morphological classification is beach slope (Fig. 2). However, the region over which beach slope is taken has an influence on the value obtained, and therefore the classification of the beach. For example, according to Wright and Short (1984), a dissipative beach is classified based on \( \tan \beta = 0.03 \) within the swash zone, and \( \tan \beta = 0.01 \) across the inshore profile, or nearshore region (Fig. 1). A reflective beach is defined as one with \( \tan \beta = 0.1 \) to 0.2 within the swash zone, and \( \tan \beta = 0.01 \) to 0.02 across the inshore profile (Fig. 2). However, Nielsen and Hanslow (1991) suggested that the distinction between dissipative or reflective should be \( \tan \beta = 0.10 \). The method of determining slope was not specified.

The presence of an offshore bar and trough, a ridge-runnel system, rip channels or beach cusps, etc., would result in an intermediate beach type between dissipative and reflective. Therefore, based on Wright and Short (1984), many beaches fall within the intermediate category. In addition, beach types can change with time, e.g., during calm and storm conditions, and at different tidal stages. Holman and Sallenger (1985) suggested that sometimes a beach can be considered as reflective during high tide and intermediate during low tide due to the different intensity of bar influence on wave breaking.

In contrast to the importance of beach slope for determining the morphodynamic classification of beaches and its influence on hydrodynamics, Douglass (1992) suggests the slope term should not be used for the formulation of runup. After an analysis of the effectiveness of including slope for estimating runup on beaches using Holman (1986) data, Douglass found the prediction of runup was just as accurate with the omission of the
slope term. He also suggested that because the slope is a dependent variable and varies significantly over time, determining its value a priori is difficult and unnecessary.

Figure 2. Beach-type classification based on beach morphology and slope (Wright and Short, 1984).
Almost all the above field studies focused mainly on the hydrodynamics of wave runup with little discussion of the corresponding morphologic response, particularly the upper limit of beach changes, which should be closely related to the wave runup. In contrast to numerous studies on wave runup itself, limited data are available relating wave runup excursion with the resulting morphologic change. In other words, it is not well-documented how the limit of wave runup is related to the limit of beach change.

In the present study, data from the prototype-scale laboratory experiments, including those conducted at SUPERTANK (Kraus et al., 1992; Kraus and Smith, 1994) and the Large-Scale Sediment Transport Facility (LSTF) (Hamilton et al., 2001; Wang et al., 2002), are analyzed to quantify the limit of wave runup and corresponding limit of beach and dune erosion. Specifically, this study examines 1) the levels of swash run-up and wave setup; 2) time-series beach-profile changes under erosional and accretionary waves; 3) the relationship between the above two phenomena; and 4) the accuracy of existing wave runup prediction methods. A new empirical formula predicting the limits of wave runup based on breaking wave height, which also infers upper limit of beach change, is based on the prototype-scale laboratory data. The formula predicting maximum wave runup is $R_{\text{max}}=H_b$. This first-order estimate has the advantage of giving zero runup for zero wave height, in a much simpler form than many existing predictive formulas.
Methods

SUPERTANK and LSTF Experiments

Data from two large-scale movable-bed laboratory studies, SUPERTANK and LSTF experiments (Fig. 3), are examined to quantify the upper limits of beach-profile change and wave runup, as well as to determine the relationship between the two. Both experiments were specifically designed to measure detailed processes of sediment transport and morphological change under varying prototype wave conditions. Dense instrumentation in the well-controlled laboratory setting allows for the detailed and accurate measurements of hydrodynamic conditions and time-series morphological changes. In addition, large-scale physical analog models such as SUPERTANK and LSTF are particularly useful in directly applying the same conditions to the real-world, without the need for scaling. SUPERTANK was conducted in a two-dimensional wave channel with beach changes caused primarily by cross-shore processes. LSTF is a three-dimensional wave basin, with both cross-shore and longshore sediment transport inducing beach change.

SUPERTANK is a multi-institutional effort sponsored by the U.S. Army Corps of Engineers, conducted at the O.H. Hinsdale Wave Research Laboratory at Oregon State University from July 29 to September 20, 1991. This facility is the largest wave channel in the United States containing a sandy beach having the capability of running
experiments comparable to the magnitude of naturally occurring waves (Kraus et al. 1992). The SUPERTANK experiment measured total-channel hydrodynamics and sediment transport along with resulting beach-profile changes. The wave channel is 104 m long, 3.7 m wide, and 4.6 m deep (with the still water level typically 1.5 m below the top) with a constructed sandy beach extending 76 m offshore (Fig. 3 upper).

Figure 3. The SUPERTANK experiment (upper) and LSTF (lower) during wave runs.
The beach was composed of 600 m$^3$ of fine, well-sorted quartz sand with a median size of 2.2 x 10$^{-3}$ m and a fall speed of 3.3 x 10$^{-2}$ m/s. The wave generator and wave channel were equipped with a sensor to absorb the energy of waves reflected from the beach (Kraus and Smith, 1994). The water-level fluctuations were measured with 16 resistance and 10 capacitance gauges. The 16 resistance gauges recorded water levels and were mounted on the channel wall, spaced 3.7 m apart, extending from near the wave generator to a water depth of approximately 0.5 m. The 10 vertical capacitance wave gauges recording wave transformation and runup were mobile with spacing varying from 0.6 to 1.8 m, and extended from the most shoreward resistance gauge to the maximum runup limit, sometimes submerged or partially buried. The 26 gauges provided detailed wave propagation patterns, especially in the swash zone.

The beach profile was surveyed following each 20- to 60-min wave simulation. The initial profile was constructed based on the equilibrium beach profile developed by Dean (1977) and Bruun (1954) as:

$$h(x) = Ax^{2/3}$$

where $h$ = still-water depth, $x$ = horizontal distance from the shoreline, and $A$ = a shape parameter corresponding to a mean grain size of 3.0 x 10$^{-3}$ m. The profile shape parameter “A” varies with both sediment grain size and fall velocity, carrying dimensions of length to the 1/3 power. The initial beach was built steeper with a greater $A$-value to ensure adequate water depth in the offshore area (Wang and Kraus, 2005). For efficiency, most SUPERTANK tests were initiated with the final profile of the previous simulation. Approximately 350 beach-profile surveys were made with an auto-tracking, infrared Geodimeter targeting prism attached to a survey rod mounted on a carriage.
pushed by researchers. Although three lines, two along the wave-channel wall and one in the center, were surveyed, only the center line was examined in this study, as the surveys showed good cross-tank uniformity. Wave-processing procedures are discussed in Kraus and Smith (1994). To separate incident-band wave motion from low-frequency motion, a non-recursive, low-pass filter was applied to the total wave record during spectral analysis. The period cutoff for the filter was set to twice the peak period of the incident waves.

The LSTF is a three-dimensional wave basin located at the U.S. Army Corps of Engineers Coastal and Hydraulic Laboratory in Vicksburg, Mississippi. Details of the design and test procedures are discussed in Hamilton et al. (2001). The LSTF was designed to study longshore sediment transport (Wang et al., 2002). Similar to the SUPERTANK experiments, the LSTF is capable of generating wave conditions comparable to the naturally occurring wave heights and periods found along low-energy coasts. The LSTF has effective beach dimensions of 30-m cross-shore, 50-m longshore, and walls 1.4 m high (Fig. 3 lower). The beach was typically designed in a trapezoidal plan shape composed of approximately 150 m$^3$ of very well-sorted fine quartz sand with a median grain size of 1.5 x 10^{-3} m and a fall speed of 1.8 x 10^{-2} m/s. Initial construction of the beach was also based on the equilibrium shape (Eq. 8). The beach profiles were surveyed using an automated bottom-tracking profiler capable of resolving bed ripples. The profiles surveyed in the center of the test basin are used here. The beach was typically replenished after three to nine hours of wave activity. Long-crested and unidirectional irregular waves with a relatively broad spectral shape were generated at a 10 degree incident angle. The wave height and peak wave period were measured with
capacitance wave gauges sampling at 20 Hz, with statistical wave properties calculated by spectral analysis. The experimental procedures in LSTF are described in Wang et al. (2002).

Data Analysis

After inspection of all 20 SUPERTANK tests, 5 tests with a total of 30 wave simulations, or wave runs, were selected for analysis in the present study. The selection was based on the particular purpose of the wave run, the trend of net sediment transport, and measured beach change. Time-series beach-profile changes and cross-shore distribution of wave height and mean water level were analyzed. During the 20 tests run during the SUPERTANK experiment, approximately 350 beach-profile surveys were conducted. For each profile, the elevation relative to mean water depth was plotted on the vertical y-axis in meters and the distance offshore on the horizontal x-axis in meters. The upper limits (UL) for the beach profile cases were identified based on the upper profile convergence point, above which no beach change occurred.

The other location of importance along the profile was the lower limit of beach change, sometimes referred to as the depth of closure. The lower limit, or profile convergence point, was identified at the depth contour below which no beach-profile change occurred. The depth of closure for a given or characteristic time interval is the most landward depth seaward of which there is no significant change in bottom elevation and no significant net sediment exchange between the nearshore and the offshore (Kraus, et al., 1999; Wang and Davis, 1999).
For all 20 SUPERTANK cases examined, the water levels and zero-moment wave heights were analyzed, where the mean water level or wave height (y-axis) was plotted against the horizontal distance offshore (x-axis), respectively. The maximum runup ($R_{\text{max}}$) was defined by the location and beach elevation of the swash gauge that contained a value larger than zero wave height, i.e., water reached that particular gauge. It is important to note that the value of $R_{\text{max}}$ is not a statistical value, but rather an actual measurement from the experiment. There may be some differences in the runup measured in this study as compared to the video method (e.g., Holland et al., 1995) and horizontally elevated wires (e.g., Guza and Thornton, 1982); however, because of the finite spacing of the capacitance wave gauges in the swash zone, the differences are not expected to be significant. Field measurements of wave runup have typically been conducted with video imagery and/or resistance wire generally 5 to 20 cm above and parallel to the beach face. Holman and Guza (1984) and Holland et al. (1995) compared these two data collection methods concluding that they are comparable in producing accurate results.

Cross-shore distribution of wave heights, or wave-energy decay, was also examined. The wave breaker point was defined at the location with a sharp decrease in wave height (Wang et al., 2002). Although the entire SUPERTANK dataset was available for this study and were analyzed, 5 cases were selected from the 20 initial test runs. The selection was based on the particular purpose of the wave run, the trend of net sediment transport, and measured beach change.

Two LSTF experiments, one conducted under a spilling breaker and one under a plunging breaker, are examined in this study. The LSTF analysis focused mainly on the
upper limit of beach change. The maximum runup was not directly measured due to the lack of swash gauges. The main objective of the LSTF analysis was to apply the SUPERTANK result to a three-dimensional beach.

The dataset from Holman (1986) was digitized. Similar statistical analysis, i.e., data normalization and linear regression analysis, were conducted to evaluate the “goodness-of-fit” between the predicted and measured runup. In addition, polynomial and exponential regression analyses were also conducted to examine potential non-linear relationships between the measured wave runup and various parameters, such as wave height, wave period, and beach slope.
Results

Overall, 30 SUPERTANK wave runs and two LSTF wave runs were analyzed (Table 1). The thirty SUPERTANK cases were composed of twelve erosional random wave runs, three erosional monochromatic wave runs, seven accretionary random wave runs, three accretionary monochromatic wave runs, and five dune erosion random wave runs, summarized in Table 1. The first two numbers in the Wave Run ID “10A_60ER” indicate the major data collection test, the letter “A” (arbitrary nomenclature) indicates a particular wave condition, and “60” describes the minutes of wave action. In design of the experiment (Kraus et al., 1992) the erosional and accretionary cases were designed based on the Dean number $N$,

$$ N = \frac{H_{bs}}{wT} $$

(9)

where $H_{bs} =$ significant breaking wave height; $w =$ fall speed of the sediment, and $T =$ wave period.

Because the SUPERTANK experiments were designed to examine the processes-response at a general beach-dune environment, rather than simulating a particular realistic setting, e.g., a certain storm at a certain beach, scaling is not a particular concern. Of course, caution should be taken when applying the SUPERTANK findings to different conditions in the real world. However, this holds true when applying findings of any
field experiment, e.g., from the Pacific coasts to the Gulf of Mexico coasts, from one setting to another.

Table 1. Summary of Selected Wave Runs and Input Wave and Beach Conditions (Notation is explained at the bottom of the table).

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<thead>
<tr>
<th>Wave Run ID</th>
<th>Ho  m</th>
<th>Tp  s</th>
<th>Lo  m</th>
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<th>N</th>
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ER = erosional random wave; EM = erosional monochromatic wave; AR = accretionary random wave; AM = accretionary monochromatic wave; DE = dune erosion case; Ho = offshore wave height; Tp = peak wave period; Lo = offshore wavelength; n = spectral peakness; N = Dean Number; Hbs = significant breaking wave height; βs = beach slope defined as the slope of the section approximately 1 m landward and 1 m seaward of the shoreline; ξ = surf similarity parameter; Hb_h = incident band wave height at the breaker line; Hb_l = low frequency band wave height at the breaker line; Hsl_h = incident band wave height at the shoreline; Hsl_l = low-frequency band wave height at the shoreline; N/C = Not calculated.
Beach Profile Change

Typically, erosion is defined as a net offshore transport of sand resulting in a net loss of beach volume above the mean water line, or shoreline. Accretion is defined as a total net onshore transport of sand, building the beach above mean water level. These definitions are applied in this study. For the SUPERTANK wave runs examined, the Dean number $N$ was typically between 5 and 10 for the erosional cases. For the accretionary cases, the $N$ was typically between 1.5 and 3 (Table 1). For the LSTF experiment, the spilling breaker case had a Dean Number of 10, indicating a highly erosive wave. A smaller Dean Number of 4.4 corresponds to the plunging breaker case, indicative of a slightly more accretionary trend (Table 1).

The first SUPERTANK wave run, ST_10A, was conducted over the monotonic initial profile (Eq. 8). Significant beach-profile change occurred with substantial shoreline recession, along with the development of an offshore bar. Figure 4 illustrates four time-series beach-profiles surveyed at initial, 60, 130, and 270 minutes. Initially, the overall foreshore exhibited a convex shape while the end profile was concave. After 60 min of wave action, considerable erosion occurred in the vicinity of the shoreline. The sand was transported offshore and deposited in the form of a bar. This trend continued during the subsequent wave runs, with additional erosion near the shoreline and further accretion of the bar with offshore directed migration. After 270 min, the bar had moved 4 m further offshore, compared to the 60-minute bar location. The 270-min profile was substantially steeper near the shoreline than the initial profile. The maximum beach-face recession occurred at the $+0.37$ m contour line. The morphology of a section of the beach profile located landward of the trough (from 15 to 18 m) did not experience any changes,
indicating that the sediment eroded from near the shoreline was transported past this section of the surf zone and deposited on the bar. The upper limit of beach-profile change can be readily identified; for this case, the upper limit was 0.66 m above mean water level (MWL) for all three time segments. There is also an apparent point of profile convergence in the offshore area, at -1.35 m depth contour, beyond which regular profile elevation change cannot be identified.

Figure 4. The first SUPERTANK wave run, ST_10A erosive case. Substantial shoreline erosion occurred on the initial monotonic profile with the development of an offshore bar. The horizontal axis “distance” refers to the SUPERTANK coordinate system and not directly related to morphological features.
The subsequent wave runs were conducted over the final profile of the previous wave run, i.e., a barred beach. The beach-profile changes are detectable, but subtle, especially for the accretionary wave cases with lower wave heights. Figure 5 shows an example of an accretionary wave run, ST_30A. Overall, compared to the initial run (Fig. 3), the beach-profile changes were much more subtle. The top of the offshore bar was eroded with sediment deposition along the landward slope of the bar (Fig. 5 upper). The subtle profile change near the shoreline, viewed at a smaller scale (Fig. 5 lower), indicates slight accretion. The accreted sand apparently came from the erosion just below the MWL. Due to the slight change, the identification of the upper limit is more difficult than the first case, and is less certain. This upper limit is determined to be at 0.31 m above MWL (Fig. 5 lower). One of the surveys (60 min.) exhibits some changes above that convergence level, however these changes have been interpreted as survey error. The offshore-profile convergence point was determined to be located at a depth contour of -1 m.

A scarp developed during some of the erosional wave runs (Fig. 6). The development of the scarp is induced by waves eroding the base of the dune or the dry-beach, subsequently causing the weight of the overlying sediment to become unstable and collapse. The resulting beach slope immediately seaward of the scarp on the beach face tends to be steeper than on a non-scarped beach. The upper limit of beach change is apparently at the top of the scarp, controlled by the elevation of the beach berm or dune, and does not necessarily represent the vertical extent of wave action. The nearly vertical scarp also induces technical difficulties of measuring the runup using wire gages. The upper limit of beach change in this study was selected at the base of the near-vertical
scarp, measured at 0.31 m above MWL during ST_60A (Fig. 6). The collapsed sediment from the scarp was deposited just below MWL (40 and 60 min). Therefore, for the scarped case, the upper limit is controlled by both wave action and gravity-driven sediment collapsing. Few beach-profile changes were measured offshore.

SUPERTANK experiments also included several cases conducted under monochromatic waves. Overall, beach-profile changes induced by monochromatic wave action were substantially different from those under the more realistic random waves (Fig. 7). The monochromatic waves tended to create irregular and undulating profiles. For ST_10, the upper limit of beach-profile change was estimated at around 0.50 m and varied slightly during different wave runs. A persistent onshore migration of the offshore bar also occurred throughout the wave runs, with several secondary bars developed in the mid-surf zone. The erratic profile evolution did not seem to approach a stable equilibrium shape, and does not have an apparent profile convergence point. In addition, it is important to note that the profile shape developed under monochromatic waves does not represent profiles typically measured in the field (Wang and Davis, 1998). This implies that morphological changes measured in movable-bed laboratory experiments under monochromatic waves may not be applicable to real world conditions, despite the easier hydrodynamic parameterization and analysis of monochromatic waves.
Figure 5. The SUPERTANK ST_30A accretionary wave run. There was subtle shoreline accretion, with an onshore migration of the offshore bar (upper). The subtle accretion near the shoreline is identified when viewed at a smaller scale (lower).
Figure 6. The SUPERTANK ST_60A dune erosion wave run. A nearly vertical scarp was developed after 40 minutes of wave action, with the upper limit of beach change identified at the toe of the dune-scarp.

Similar analyses as described above were also conducted to a set of three-dimensional laboratory movable-bed experiments at LSTF. The waves generated in LSTF had smaller wave heights as compared to the SUPERTANK waves. Two cases with distinctively different breaker types, one spilling and one plunging, were examined in this study. The beach profiles used in the following discussion are averaged over the middle section of the wave basin, and therefore represent the average condition of the 3-dimensional beach.
The spilling wave run was initiated with the Dean equilibrium beach profile. Because of the smaller wave heights, the beach-profile changes occurred at a slower rate. Similar to the first SUPERTANK wave run, ST_10A, a subtle bar formed over the initial monotonic beach profile (Fig. 8 upper). The upper limit of beach change was 0.23 m above MWL under the spilling waves, as identified from the smaller scale plot (Fig. 8 lower). Sediment from the eroding foreshore was apparently transported seaward to the offshore bar. As the foreshore eroded, the offshore bar accumulated sediment increasingly with each subsequent wave run. Between 5 and 9 m offshore, little to no
change occurred which could suggest that the sediment bypassed this region and terminated transport at the offshore bar. The profile converges on the seaward slope of the offshore bar.

Figure 8. The LSTF Spilling wave case. Erosion occurred in the foreshore and inner surf zone. The eroded sediment was deposited on an offshore bar.
For the plunging wave case at LSTF, shoreline advancement occurred with each wave run along with a persistent onshore migration of the bar (Fig. 9 upper). This corresponds to a lower Dean number of 4.4. The accumulation at the shoreline was subtle, but can be identified when viewed at a smaller spatial scale (Fig. 9 lower). The upper limit of beach-profile change is located at 0.26 m above MWL, with the lower limit identified at the profile convergence point midway on the seaward slope of the bar. Also, persistent erosion was measured just below the MWL and at a region landward of the initial secondary bar. The eroded sediment landward of the trough may have contributed to the onshore migration of the offshore bar (Fig. 9 upper). Overall, the trends observed in the three-dimensional LSTF experiment were comparable to those in the two-dimensional SUPERTANK experiment.

Table 2 summarizes the upper and lower limits of change during each wave run, including the breaking wave height. Overall, for the 30 SUPERTANK wave runs and 2 LSTF wave runs, the incident breaking wave height ranged from 0.26 to 1.18 m (Table 2). The measured upper limit of profile change, including the scarped cases, ranged from 0.23 to 0.70 m. The lower limit of beach change ranged from 0.50 to 1.61 m below MWL. Relationships between the profile changes and wave conditions are discussed in the following sections.
Figure 9. The LSTF Plunging wave case. Slight foreshore accretion and landward migration of the offshore bar occurred during the wave run.
Table 2. Summary of Breaking Wave, Maximum Wave Runup, Upper and Lower limits of Beach-Profile Changes, and Presence of a Scarp for Each Analyzed Wave Run.

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<th>$U_L$ m</th>
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<td><strong>LSTF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spilling</td>
<td>0.26</td>
<td>N/M</td>
<td>0.23</td>
<td>0.62</td>
<td>No</td>
</tr>
<tr>
<td>Plunging</td>
<td>0.27</td>
<td>N/M</td>
<td>0.26</td>
<td>0.5</td>
<td>No</td>
</tr>
</tbody>
</table>

$H_b =$ breaker height measured; $R_{max} =$ maximum runup measured (not statistical); $U_L$, $L_L =$ upper and lower limit of beach change, respectively; $NM =$ not measured.
Cross-shore Distribution of Wave Height

The wave-height decay is representative of the wave-energy dissipation as a wave propagates onshore. Detailed wave decay patterns were measured by the closely spaced wave gauges for both SUPERTANK and LSTF experiments. Figure 10 shows time-series wave decay patterns measured at the first SUPERTANK wave run, ST_10A. As discussed earlier, considerable beach profile change, for example the formation of an offshore bar, was measured during this wave run (Fig. 4). The substantial morphology change also influenced the pattern of wave decay.

Figure 10. The cross-shore random wave-height distribution measured during SUPERTANK ST_10A; yellow arrow indicates the breaker point.
The inflection point of wave decay migrated slightly as the beach morphology changed from the initial monotonic profile to a barred-beach profile. This point is defined as the location and height at which the wave breaks (Wang et al., 2002). For ST_10A, the significant breaking wave height is 0.68 m. The rate of wave-height decay tends to be smaller in the mid-surf zone (10 to 20 m) as compared to the breaker zone (20 to 25 m) and the inner surf zone (landward of 10 m). The offshore wave height remains largely constant until reaching the breaker line.

The wave decay pattern for the longer period accretionary wave case, ST_30A (Fig. 11), was considerably different than the steep erosive waves. The significant breaking wave height was 0.36 m. The time-series wave pattern remained constant for each wave run, apparently not influenced by the subtle morphology change (Fig.5). The offshore bar formed at 30 m during the previous experiment with higher wave heights. Instead of wave breaking over the bar, shoaling of the long period wave was measured (at around 30 m). The main breaker line is identified at around 20 m.
For the dune erosion case, the wave-height remained largely constant offshore, with a slight decay in wave height up to the breaker point (Fig. 12). Dramatic wave-height decay was measured over the offshore bar at approximately 30 m, with a breaker height of 0.61 m. A slight increase in wave height, possibly due to wave shoaling, was measured at around 15 m offshore, followed by a sharp decrease in wave energy.
The cross-shore distribution of wave heights for the monochromatic wave case ST_10 was rather erratic with both temporal and spatial inconsistencies (Fig. 13). This corresponds to the irregular beach-profile change observed during this wave case (Fig. 7). The breaking wave height varied considerably, from 0.72 to 0.81 m. The wave-height variation in the offshore region, seaward of the breaker line around 30 m, is probably related to oscillations in the wave tank. However, the irregular breaking wave heights were likely caused by reflection of the regular waves off the beach face.
Figure 13. The cross-shore monochromatic wave-height distribution measured during SUPERTANK ST_I0.

The LSTF experiments were designed to examine the effects of different breaker types on sediment transport and morphology change. Similar offshore wave heights of 0.27 m were generated for both cases (Fig. 14). However, the cross-shore distribution of wave heights varies, especially in the vicinity of the breaker line, where the spilling breaker wave dissipation was much more gradual than the plunging breaker. The breaking wave heights were measured at 0.26 m and 0.27 m; for the spilling and plunging waves, respectively. For the spilling wave case, the offshore bar was low with a wide
crest at around 12 m offshore (Fig. 9) with the main breaker line just seaward of the bar crest (Fig. 14). The narrow bar during the plunging case migrated onshore from 13 to 11 m (Fig. 9), with the main breaker line occurring directly over the crest of this bar (Fig. 14). Again, the wave height decay at the breaker line was more abrupt for the plunging case than for the spilling case, as expected.

Figure 14. The cross-shore wave-height distribution and $\gamma$ for the LSTF Spilling and Plunging wave cases.
Wave Runup

The extent and elevation of wave runup for the SUPERTANK experiments were measured directly by the closely spaced swash gages (Kraus and Smith, 1994). Figure 15 shows the cross-shore distribution of time-averaged water level for the erosive wave run, ST_10A. As expected, the mean water level in the offshore area remained largely at zero. Elevated water levels were measured in the surf zone. It is important to separate the elevation caused by wave setup and swash runup, although it is often difficult to do. Currently, there is no widely accepted method for separating the setup from the swash in total wave runup. In this study, the elevated water levels seaward and landward of the still-water shoreline are regarded as wave setup and swash runup, respectively. This convention coincides well with the inflection point of the change in slope of the cross-shore distribution of the mean water level (Fig. 15). The setup approaches the shoreline asymptotically following the slope or curvature of the beach face, whereas the swash appears to be nearly vertical controlled by the gauge elevation. For this case, the setup measured at the still-water shoreline was 0.1 m, which is about 17 percent of the total wave runup of 0.6 m, consistent with the approximation for setup by Guza and Thornton (1982). The slight difference in wave runup among different wave runs was attributed to the considerable beach-profile change during the first SUPERTANK wave case.
Figure 15. Wave runup measured during SUPERTANK ST_10A.

For the accretionary wave case, ST_30A, the curvature in the mean-water level also coincides with the still-water shoreline (Fig. 16). However, considerable variations in the wave setup occurred at the shoreline. The slight decrease in mean water level measured just seaward of 20 m offshore during the 130-min wave run is attributed to setdown. The increase measured just seaward of 20 m offshore during the 270-min wave run is likely caused by instrument error at that particular location. The average setup at the shoreline was approximately 0.07 m, also about 17 percent of the total measured wave runup of 0.42 m.
Figure 16. Wave runup measured during SUPERTANK ST_30A.

Total wave runup was significantly limited by the vertical scarp as shown in the dune erosion case of ST_60A (Fig. 17). A broad set down was measured just seaward of the main breaker line. The curvature of the cross-shore distribution of the mean water level occurred between 10 and 11 m, which was different from the still-water shoreline at 8 m. The setup measured at the water-level curvature at 11 m was approximately 0.03 m. Despite the limited total runup, the wave setup contributes 18 percent of the total wave runup of 0.17 m, similar to the above two cases.

The cross-shore distribution of time-averaged mean water levels for ST_10 were somewhat erratic (Fig. 18), similar to the erratic beach-profile changes and cross-shore distribution of wave heights for the monochromatic wave runs. Different from the irregular wave cases, a zero mean-water level was not measured at a several offshore wave gauges.
Figure 17. Wave runup measured during SUPERTANK ST_60A.

Figure 18. Wave runup measured during SUPERTANK ST_I0.
In addition, considerable variances among different wave runs were also measured. The total wave runup varied from 0.16 to 0.35 m, with an average of 0.26 m. The curvature in the mean-water level distribution coincides with the still-water shoreline at 5 m. The maximum setup at the still-water shoreline was 0.08 m, which is 31 percent of the total wave runup. The lesser contribution of the swash runup to the total wave runup can be attributed to the lack of low-frequency motion in the monochromatic waves, exemplified in Table 1, in which the low-frequency components of the swash (Hsl_l) for monochromatic waves were much smaller than the contribution for random waves.
Discussion

Relationship between Wave Runup, Incident Wave Conditions, and Limit of Beach Profile Change

The measured breaking wave height, upper limit of beach-profile change, and wave runup from the SUPERTANK experiments are compared in Figure 19. The thirty cases examined are divided into three categories, including non-scarped random wave runs, scarped random wave runs, and monochromatic wave runs. The upper limit of beach change was almost equal to the maximum excursion of wave runup for the non-scarped cases. In addition, the maximum wave runup was nearly equal to the breaking wave height for the non-scarped random wave cases as well. This suggests that the breaking wave height is equal to the maximum elevation of wave runup which is equal to the upper limit of beach change, emphasized by the 16 non-scarped random wave runs, except the three cases 10B_20ER, 10E_270ER and 30D_40AR. The cause of the discrepancy in the data for these outliers varies. During the SUPERTANK experiment, it was confirmed that the capacitance gauges could record the wet sand or water surface if the sand was fully saturated. Some degradation in the response was found if the sand was not fully saturated, which could explain the limited swash runup measurement during the first 20 min run of ST10B. However, a direct cause for the other two questionable measurements is not clear.
Figure 19. Relationship between breaking wave height, upper limit of beach profile change, and wave runup for the thirty SUPERTANK cases examined.

Based on the criteria of identifying the upper limit of beach change at the toe of the scarp, for the scarped random wave cases the breaking wave height was much higher than the elevation of wave runup, which was limited by the vertical scarp. The definition was based on the fact that any change occurring above this point was likely due to the weight of the overlying sediment collapsing. The true upper limit of change is thus controlled by the elevation of the beach or dune. No relationship among the breaker height, wave runup, and beach-profile change was found for the scarped random wave
runs. However, it is important to note that the upper limits of beach changes for scarped cases are not expected to relate to wave runup.

The much lower wave runup under monochromatic waves as compared to the breaker height was found to be caused by the lack of low-frequency motion. Baldock and Holmes (1999) suggested that simulating irregular waves with overlapping monochromatic swash events could reproduce both low and high frequency spectral characteristics of the swash zone. No SUPERTANK experiments were designed to investigate this. No relationship could be found among the above three factors for monochromatic waves. This further suggests that despite the convenience associated with studying the hydrodynamics associated with regular waves, including monochromatic waves in laboratory experiments related to morphology changes, they do not have direct real-world applications.

Based on the above observations from the SUPERTANK data (Fig. 19), a direct relationship between the measured runup height on a non-scarp beach and the breaker height is proposed:

$$R_{\text{max}} = H_{bs}$$

(10)

The average ratio of $R_{\text{max}}$ over $H_{bs}$ for the 16 non-scarped wave cases was 0.93 with a standard error on the mean of 0.05. Excluding the three outliers, 10B_20ER, 10E_270ER and 30D_40AR, the average $R_{\text{max}}/H_{bs}$ was 1.01 with a standard error of 0.02. To be conservative due to the limited data coverage, a value of one is used in Eq. (10). In addition, the maximum runup elevation for the non-scarped random wave cases was approximately equal to the upper limit of beach change,

$$R_{\text{max}} = U_L$$

(11)
This is important for relating morphological changes to wave runup on beaches, which is a main goal of runup studies. However, this relationship is most directly applied to wave similar to those generated in the two physical analog models. In addition, a wider range of runup and significant wave heights would likely increase the reliability of the correlation coefficient.

Therefore, it is suggested that the breaker height is equal to the upper limit of wave runup which is equal to the upper limit of beach changes,

\[ H_{br} = R_{max} = U_L \]  

(12)

Comparisons of the measured wave runup with the various existing empirical formulas (Eqs. 4, 6, and 7) and the new model (Eq. 10) are summarized in Figure 20 and Table 3. As shown in Fig. 20, the simple new model reproduced the measured wave runup much closer than the other formulas. For the 16 non-scarped SUPERTANK wave cases, 81% of the predictions from Eq. (10) fall within 15% of the measured wave runup. In contrast, for Eqs. (4), (6) and (7), only 25%, 6% and 13% of the predictions, respectively fall within 15% of the measured values. Eqs. (6) and (7) under-predicted the measured wave runup for the erosional cases and over-predicted runup for the accretionary wave cases. The loss of predictive capability is caused by the substantially greater \( \xi \) for the gentle long-period accretionary waves than for the steep short-period erosional waves (Table 1). Agreement between measured and predicted values was actually reduced by including the surf similarity parameter, \( \xi \). The simpler Eq. (4), using just offshore wave height, more accurately reproduced the measured values of wave runup than Eqs. (6) and (7).
Figure 20. Comparison of measured and predicted wave runup for Eqs. (10), (4), (6), and (7).

Table 3. Summary of Measured and Predicted Wave Runup Equations. The bold font indicates predicted values that fall within 15% of the measured runup.

<table>
<thead>
<tr>
<th>Wave Run</th>
<th>$H_{bs}$ m</th>
<th>$R_{max}$ m</th>
<th>Eq 4 m</th>
<th>Eq 6 M</th>
<th>Eq 7 m</th>
<th>Eq 10 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10A_60ER</td>
<td>0.68</td>
<td>0.60</td>
<td>0.59</td>
<td>0.43</td>
<td>0.31</td>
<td>0.68</td>
</tr>
<tr>
<td>10A_130ER</td>
<td>0.68</td>
<td>0.60</td>
<td>0.59</td>
<td>0.40</td>
<td>0.29</td>
<td>0.68</td>
</tr>
<tr>
<td>10A_270ER</td>
<td>0.68</td>
<td>0.70</td>
<td>0.59</td>
<td>0.43</td>
<td>0.31</td>
<td>0.68</td>
</tr>
<tr>
<td>10B_20ER</td>
<td>0.65</td>
<td>0.33</td>
<td>0.54</td>
<td>0.49</td>
<td>0.38</td>
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</tr>
<tr>
<td>10B_60ER</td>
<td>0.67</td>
<td>0.70</td>
<td>0.55</td>
<td>0.42</td>
<td>0.31</td>
<td>0.67</td>
</tr>
<tr>
<td>10B_130ER</td>
<td>0.69</td>
<td>0.64</td>
<td>0.55</td>
<td>0.36</td>
<td>0.26</td>
<td>0.69</td>
</tr>
<tr>
<td>10E_130ER</td>
<td>0.72</td>
<td>0.77</td>
<td>0.52</td>
<td>0.53</td>
<td>0.46</td>
<td>0.72</td>
</tr>
<tr>
<td>10E_200ER</td>
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<td>0.78</td>
<td>0.52</td>
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</tr>
<tr>
<td>10E_270ER</td>
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<td>0.45</td>
<td>0.52</td>
<td>0.47</td>
<td>0.41</td>
<td>0.76</td>
</tr>
<tr>
<td>30A_60AR</td>
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<td>0.41</td>
<td>0.28</td>
<td>0.70</td>
<td>0.72</td>
<td>0.41</td>
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<tr>
<td>30A_130AR</td>
<td>0.39</td>
<td>0.43</td>
<td>0.27</td>
<td>0.64</td>
<td>0.66</td>
<td>0.39</td>
</tr>
<tr>
<td>30A_200AR</td>
<td>0.41</td>
<td>0.42</td>
<td>0.28</td>
<td>0.64</td>
<td>0.66</td>
<td>0.41</td>
</tr>
<tr>
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<td>0.25</td>
<td>0.67</td>
<td>0.73</td>
<td>0.40</td>
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<tr>
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<td>0.25</td>
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<td>0.79</td>
<td>0.39</td>
</tr>
<tr>
<td>30C_270AR</td>
<td>0.39</td>
<td>0.42</td>
<td>0.25</td>
<td>0.73</td>
<td>0.79</td>
<td>0.39</td>
</tr>
<tr>
<td>30D_40AR</td>
<td>0.42</td>
<td>0.23</td>
<td>0.30</td>
<td>0.69</td>
<td>0.76</td>
<td>0.42</td>
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</table>
Measured wave runup from the SUPERTANK experiment was compared with the predicted runup. For the predictive formula proposed in this study, the linear relationship between breaking wave height and wave runup was found to be improved with the exclusion of the three questionable measurements (Figs. 21 and 22). The correlation coefficient was nearly three times better for the dataset without the anomalous values than for the analysis with the anomalies, with a $R^2$ of 0.92 and 0.32 respectively. The proportionality coefficient from the linear regression supports the equation suggested by this study.

$$y = 1.0563x$$

$R^2 = 0.3117$

Figure 21. Measured versus predicted runup for Eq. (10), including the anomalies.
A linear regression analysis for Eq. (4), presented by Guza and Thornton (1982), also suggested a linear relationship with wave height (also excluding the three anomalies), with a correlation coefficient of $R^2 = 0.76$ (Fig. 23). The predicted runup from Eq. (7), Stockdon et al., (2006), was analyzed with the measured runup, minus the three anomalies (Fig. 24). The $R^2$ was 0.60, with a large intercept of 1.13. The two clusters in the data illustrate the over and under prediction of wave runup by Eq. (7) for the erosional and accretional waves, respectively. The predicted versus measured runup presented by Holman (1986), Eq. (6), was also analyzed through a linear regression (Fig. 25). A similar trend as the predicted runup from Eq. (7) was observed, with a slightly reduced $R^2$ of 0.52. The two clusters in the data again illustrate an over and under prediction of the erosive and accretionary waves.
Figure 23. Measured versus predicted runup for Eq. (4), excluding the anomalies.

Figure 24. Measured versus predicted runup for Eq. (7), excluding the anomalies.
Holman (1986) data were digitized from the original paper. The purpose was to reproduce some of his analyses. The Figure 4c in Holman (1986) is reproduced in Figure 26 below, showing the relationship between runup and wave height, without inclusion of setup. The original dataset from Holman (1986) is shown above the digitized data to ensure the figure was accurately reproduced. Two linear regression analyses were conducted, one with a forced zero-intercept and one without. The linear regression without a forced zero intercept resulted in a large intercept of 1.23 with a $R^2$ of 0.45. However, the forced zero-intercept linear regression resulted in a $R^2$ of 0.36. Both suggest that the direct relationship between wave runup and wave height does not exist. It is worth noting that the swash runup is compared to significant offshore wave height.
measured at 8 m water depth, and not to breaker height. As shown in Figure 26, for the same wave height, a large variation of wave runup values were measured, which does not seem to be reasonable. It is not clear how and why a 1 m wave induced a 2.5 m swash runup, while a 4 m wave induced less than 2 m of swash runup. It is questionable as to how a 1 m offshore wave can induce a greater swash runup as compared to a 4 m offshore wave.

The Figure 7c in Holman (1986) showing the relationship between surf similarity and swash runup (minus setup) is reproduced in Figure 27. The measured swash runup was normalized with offshore wave height. A linear regression analysis was conducted, resulting in a $R^2$ of 0.62. An exponential fit was also attempted (Figure 27). The correlation coefficient was improved slightly, 0.69 versus 0.62, as compared to the linear fit.
Figure 26. Figure 4c from Holman (1986) comparing swash runup (minus setup) and offshore significant wave height (above); the digitized data from Figure 4 (Holman, 1986) with two linear regression analyses (below).
Figure 27. Figure 7c from Holman (1986) comparing swash runup (minus setup) and the surf similarity parameter (above); the digitized data from Figure 4 (Holman, 1986) with a linear and exponential regression analysis (below).
Recall, Douglass (1992) re-analyzed the Holman (1986) data set used in Eq. (6) and found no correlation between runup and beach-face slope. Douglass further argued that beach slope is a dependent variable free to respond to the incident wave energy and should not be included in a runup prediction. In practice, beach face slope is a difficult parameter to define and determine. Except for Stockdon et al. (2006), a clear definition of beach slope is not given in most studies. However, the Stockdon et al. (2006) definition of beach slope is very specific for video measurements and is difficult to utilize for other methods. In the present study, the slope was defined over that portion of the beach extending roughly 1 m landward and seaward from the still-water shoreline. Based on this definition, the Holman (1986) dataset overall had a \( \tan \beta = 0.08 \) to 0.17 across the swash zone; the SUPERTANK dataset had a \( \tan \beta = 0.09 \) to 0.15 across the swash zone. The resulting beach-profiles from SUPERTANK under accretionary waves tended to have slopes of 0.13 to 0.20, whereas the beach-profiles from erosional waves had slopes of 0.08 to 0.14 (Table 1). Nielsen and Hanslow (1991) also discussed the difficulty in defining beach slope, whether across the beach face or surf zone, with the presence of bars on intermediate beaches further complicating the measurement of beach slope. Including the beach slope thus adds ambiguity in applying the empirical formulas. In addition, Douglass (1992) omitted the beach slope term from the Holman (1986) dataset, arguing that the wave steepness term in the surf similarity parameter was responsible for \( \xi \)'s predictive capability, rather than the inclusion of slope. This further suggests that the use of the slope term in runup predictions not only induces more uncertainties, but is also generally unnecessary.
However, the inclusion of slope may be required under certain circumstances. For a very gentle beach, a large swash excursion is associated with a fixed runup, as compared to a steep beach. It is reasonable to argue that if infiltration, saturation, and bottom friction are significant, beach slope cannot be neglected. However, this is beyond the scope of this study and is also not the case for most of the existing studies reviewed (Guza and Thornton, 1982; Nielsen and Hanslow, 1991; Hanslow and Nielsen, 1993; Ruggiero et al., 1996; Stockdon et al., 2006). Some experiment data have shown that the friction loss due to wave propagation is small (Komar, 1998).

Determining offshore wave height may also cause uncertainty. In most field studies, the offshore wave height was taken to be the value measured at a wave gage in the study area. Similarly, here it is taken as that from the offshore-most gauge. Under extreme storm conditions in which wave gauges often fail, estimating the offshore wave height may not be straightforward (Wang et al., 2006). The definition of an offshore wave height varies between studies, in which it is often taken at whatever depth the instrument is deployed (Guza and Thornton, 1981; Guza and Thornton, 1982; Holman, 1986; Stockdon et al., 2006). Stockdon et al. (2006) concluded that runup was better predicted using breaking wave height when it is available; however the often lack of measured breaking wave height was argued as the reason for the use of offshore wave height for predicting runup. Theoretically, the beach slope and wave characteristics are contributing factors in the breaker height and type (Nielsen and Hanslow, 1991). Therefore, using breaking wave height may indirectly incorporate these factors.
Eq. (10) was applied to examine the three-dimensional LSTF data. The wave runup was not directly measured at LSTF. It is assumed here that the maximum runup is equal to the upper limit of beach-profile change. This is a reasonable assumption, based on the SUPERTANK data. For the spilling wave case, using the upper limit of beach change at 0.23 m as the value for total wave runup, the breaking wave height of 0.26 m resulted in an over-prediction of 13%. For the plunging wave case, the upper limit of beach change was 0.26 m, which is almost equal to the 0.27 m breaking wave height. Based on these limited results, the LSTF results support the predictive capabilities of the new formula (Eq. 10). It is worth noting that the 1.5 x 10^{-3} m (0.15 mm) sediment used in the LSTF experiments is finer than the 2.2 x 10^{-2} m (0.22 mm) used in the SUPERTANK experiment.

A Conceptual Derivation of the Proposed Wave Runup Model

Assuming a normally incident wave and neglecting longshore variations and infiltration, the major forces acting on a water element in the swash zone in the cross-shore direction, \( x \), (Fig. 28) can be balanced as:

\[
-\rho g \Delta x \Delta y \Delta z \sin \beta - f \frac{f}{8} \rho \Delta x \Delta y V_x^2 = \rho \Delta x \Delta y \Delta z \frac{\partial V_x}{\partial t}
\]

(11)

where, \( \rho \) = density of water; \( g \) = gravitational acceleration; \( \Delta x, \Delta y, \) and \( \Delta z \), = length, width, and height, of the water element, respectively; \( \beta \) = beach slope; \( f \) = a friction coefficient; and \( V_x \) = velocity. Eq. (11) can be reduced to

\[
\frac{\partial V_x}{\partial t} = -g \sin \beta - \frac{f}{8 \Delta z} V_x^2
\]

(12)
Assuming the friction force is negligible, an assumption supported by experiments discussed in Komar (1998), Eq. (12) is further reduced to

\[
\frac{\partial V_x}{\partial t} = -g \sin \beta \tag{13}
\]

Integrating Eq. (13) with respect to time, yields

\[
V_x = V_o - gt \sin \beta \tag{14}
\]

where, \(V_o\) = initial velocity. Integrating Eq. (14) again with respect to time gives the swash excursion, \(x\), as a function of time, \(t\):

\[
x(t) = V_o t - \frac{gt^2}{2} \sin \beta \tag{15}
\]

From Eq. (14), the maximum uprush occurs at a time, \(t_{max}\), when the velocity becomes zero:

\[
t_{max} = \frac{V_o}{g \sin \beta} \tag{16}
\]
with a corresponding value of maximum uprush of

\[ x(t_{\text{max}}) = \frac{V_o^2}{2g \sin \beta} \]  \hspace{1cm} (17)

Assuming a small and planar foreshore slope (Shen and Meyer, 1963; Mase, 1988; Baldock and Holmes, 1999), i.e. \( \tan \beta \approx \sin \beta \), the elevation of the maximum swash uprush \( R_{sv_{\text{max}}} \)

\[ R_{sv_{\text{max}}} = x(t_{\text{max}}) \tan \beta = \frac{V_o^2}{2g \sin \beta} \tan \beta \approx \frac{V_o^2}{2g} \]  \hspace{1cm} (18)

Eq. (18) suggests that the maximum elevation of swash runup is not a function of beach slope when friction forcing is neglected.

The initial velocity \( V_o \) can be approximated by the velocity of the wave, \( C_g \). In shallow water, the wave velocity is limited by the local water depth, \( h_i \):

\[ V_o \approx C_g = \sqrt{gh_i} \]  \hspace{1cm} (19)

Assuming a linear relationship between local breaking wave height, \( H_{bl} \), and the water depth, \( h_i \)

\[ H_{bl} = \gamma h_i \]  \hspace{1cm} (20)

where \( \gamma \) is the breaker index. Eq. (19) then becomes

\[ V_o^2 \approx C_g^2 = g \frac{H_{bl}}{\gamma} \]  \hspace{1cm} (21)

Substituting Eq. (21) into Eq. (18)

\[ R_{sv_{\text{max}}} = \frac{V_o^2}{2g} = \frac{gH_{bl}}{2g\gamma} = \frac{H_{bl}}{2\gamma} \]  \hspace{1cm} (22)
It is reasonable to assume that the initial $V_o$ can be taken at the main breaker line, using significant breaker height, $H_{bs}$. Eq. (22) becomes

$$R_{sr_{-\text{max}}} = \frac{H_{bs}}{2\gamma} = \alpha H_{bs}$$

(23)

where $\alpha = 1/2\gamma$. Eq. (23) indicates a linear relationship between breaking wave height and the maximum swash runup, supporting the findings from the SUPERTANK experiment.

Kaminsky and Kraus (1994) examined a large dataset, including both laboratory and field measurements, on breaking wave criteria. They found that the majority of the $\gamma$ values range from 0.6 to 0.8, which yields $\alpha$ values from 0.63 to 0.83. Based on the discussion of Figs. 15 through 18, the swash runup constitutes approximately 83% of the total wave runup. Adding the 17% contribution from the wave setup, then the total wave runup, $R_{max}$, is roughly proportional to the breaking wave height at unity, further supporting the new model developed from the SUPERTANK dataset.
Conclusions

The SUPERTANK data indicate that the vertical extent of wave runup above mean water level on a non-scarped beach is approximately equal to the significant breaking wave height. A simple formula for predicting the maximum wave runup, $R_{max}$: $R_{max} = H_b$, was developed and proven by a conceptual derivation. The new model was applied to the 3-dimensional LSTF experiments, and accurately reproduced the measured wave runup. By including the surf similarity parameter, as with several existing empirical formulas, the accuracy of the calculated wave runup decreased as compared to the measured values. In addition, the upper limit of beach change was found to be approximately equivalent to the maximum vertical excursion. Because the maximum runup was found to be directly related to breaker height, and the upper limit of beach change was approximately equal to the maximum runup, the breaker height can be used to assess morphological changes on beaches due to runup. In other words, $U_L = R_{max} = H_b$.

For monochromatic waves, the measured wave runup was much smaller than the breaking wave height. The lack of low-frequency modulation limits the wave runup for monochromatic waves. The hydrodynamics and beach-profiles resulting from regular waves are not comparable to the real-world. Therefore, results from studies with monochromatic waves are not useful in predicting wave runup and beach changes for natural beaches.
An exception to the direct relationship between breaking wave height, runup and upper limit of beach change concerns dune or beach scarping. The steep scarp substantially limits the uprush of swash motion, resulting in a much reduced maximum level as compared with the non-scarping cases. The nearly vertical scarp also induces technical difficulties of measuring the runup using wire gages. Most predictive equations are not expected to apply to scarped cases. Because the actual upper limit of beach change is controlled by the elevation of the backbeach or dune, the upper limit of wave runup was not found to correlate with the breaker height for these cases.

Based on the SUPERTANK and LSTF experiments, the upper limit of beach-profile change was found to be approximately equal to the maximum vertical excursion of wave runup for waves between 0.20 and 1.00 m. Therefore, the limit of wave runup can serve as an estimate of the landward limit of beach change. Physical situations that are exceptions to this direct relationship are those with beach or dune scarping. Accurately predicting the maximum elevation of wave runup was best predicted using the breaker height. The applicability of this relationship will contribute to predictive capabilities of assessing beach change for nourishment and structure designs, engineering, and many other coastal management practices.
References


Appendices
### Appendix I - Notation

The following symbols were used in this paper:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>shape parameter relating to grain size and fall velocity</td>
</tr>
<tr>
<td>$C_g$</td>
<td>wave group velocity</td>
</tr>
<tr>
<td>$f$</td>
<td>friction coefficient</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$h$</td>
<td>still-water depth</td>
</tr>
<tr>
<td>$H$</td>
<td>wave height</td>
</tr>
<tr>
<td>$H_b$</td>
<td>breaking wave height</td>
</tr>
<tr>
<td>$H_d$</td>
<td>local breaking wave height</td>
</tr>
<tr>
<td>$H_{bs}$</td>
<td>significant breaking wave height</td>
</tr>
<tr>
<td>$H_{h,h}$</td>
<td>high frequency component of wave height at the breaker line</td>
</tr>
<tr>
<td>$H_{h,l}$</td>
<td>low frequency component of wave height at the breaker line</td>
</tr>
<tr>
<td>$h_l$</td>
<td>local water depth</td>
</tr>
<tr>
<td>$H_o$</td>
<td>significant deepwater wave height</td>
</tr>
<tr>
<td>$H_{sl,h}$</td>
<td>high frequency component of wave height at the shoreline line</td>
</tr>
<tr>
<td>$H_{sl,l}$</td>
<td>low frequency component of wave height at the shoreline line</td>
</tr>
<tr>
<td>$L_L$</td>
<td>lower limit of beach change</td>
</tr>
<tr>
<td>$L_o$</td>
<td>deepwater wavelength</td>
</tr>
<tr>
<td>$n$</td>
<td>spectral peakness</td>
</tr>
<tr>
<td>$N$</td>
<td>Dean number</td>
</tr>
<tr>
<td>$R_{max}$</td>
<td>maximum swash runup</td>
</tr>
<tr>
<td>$R_s$</td>
<td>significant wave runup</td>
</tr>
<tr>
<td>$R_{sr, max}$</td>
<td>elevation of maximum swash uprush</td>
</tr>
<tr>
<td>$R_2$</td>
<td>2% exceedence of runup</td>
</tr>
<tr>
<td>$T$</td>
<td>wave period</td>
</tr>
<tr>
<td>$t_{max}$</td>
<td>time of maximum swash excursion</td>
</tr>
<tr>
<td>$T_p$</td>
<td>peak wave period</td>
</tr>
<tr>
<td>$U_L$</td>
<td>upper limit of beach change</td>
</tr>
<tr>
<td>$V_o$</td>
<td>initial velocity</td>
</tr>
<tr>
<td>$V_x$</td>
<td>velocity of a water particle in the cross-shore direction</td>
</tr>
<tr>
<td>$x$</td>
<td>cross-shore coordinate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>beach slope</td>
</tr>
<tr>
<td>$\beta_f$</td>
<td>foreshore beach slope</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>breaker index</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>length of a water particle</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>width of a water particle</td>
</tr>
</tbody>
</table>
\( \Delta z \) height of a water particle
\( \eta \) wave setup
\( \eta_M \) wave setup under monochromatic waves
\( \eta_d \) wave setup at the shoreline
\( \xi \) surf-similarity parameter
\( \rho \) water density
\( \omega \) sediment fall velocity